Second Year Examination in Arts (Economics) 2017/2018
(End of first semester)
ECN 2132: Mathematical Economics
Time Allowed: Two hours (02) only
Calculators are allowed.

Candidate should answer **13** (thirteen) questions.
All questions in **section A, 4 questions** from **section B** and **2 questions** from **section C.**

Section A: Answer all questions in this section (total marks 20)

1. Differentiate following functions
   a. $y = 6e^{1-2x}$
   b. $y = x^3 e^x$
   c. $y = \ln(2 + x^2)$

2.
   a. $f(X) = g(X) * h(x)$, where g(x) and h(x) are both differentiable functions. Derive a formula to get the derivative of f(x) with respect to x.

3.
   a. $z = f(x,y)$ (z is a function of x and y)
      $y = g(x)$ (x and y are not independent, y is a function of x)
      Derive an equation to measure the effect of change of x on z.

4.
   a. Find the total differential for the following function
      $z = 2x^2(4x - 4y)$

5. For the below function derive all second order partial derivatives and cross partial derivatives.
   a. $z = 32x^{0.5}w^{0.25}$

6. Find the integral of following functions
   a. $y = \int (2t^2 - 3t^{\frac{-1}{4}} + t)\, dt$

7. Evaluate the following definite integrals.
   a. $\int_1^4 (X^3 + X + 6)\, dx$

   b. $\int_1^8 (X^{-2/3} + X + 6)\, dx$

Section B: Answer any 4 questions from section B. (total marks 10)

Question number 8 and 9 are based on following function. State whether the following statements related to the given function are true or false. Prove your answer.

$$Y = f(x,z) = x^3 z$$

8. The function is homogeneous of degree 3.

9. Value of Y increases with the increase of z, but at a decreasing rate

10. If $Y = f(x) = x^3 - 3x^2 - 24x$. The function is concave and relative maximum when x equal 4 (x = 4) do you agree with this statement? and prove your answer.

Question number 11 to 13 are based on following Cobb Douglas production function
$$Q = AK^{\alpha}L^{\beta} \qquad \text{where A> 0. } \alpha > 0, \beta>0$$

11. Find the slope of the isoquant for above production function.

12. What happen to the slope of isoquant as K increases. (prove your answer)

13. Prove that output elasticity with respect to capital is equal to α and output elasticity with respect to labourl is equal to β.

14. Using an appropriate example, explain the difference between first order homogeneous differential equation and first order non-homogeneous differential equation.

Section C: Answer any 2 questions from section C. (total marks 30)

15. Consider a monopoly market whose Total Cost and Total Revenue functions are given below;
$$TC = 40 + 0.4Q^2$$
$$TR = 360Q - 2.1Q^2$$
     a. Determine the profit maximizing output level and price.   (4 marks)
     b. Derive the second order condition for profit maximization.   (4 marks)
     c. Assume government imposes a lump sum tax (T) on the firm. What would be the firm's total cost function after tax.   (2 marks)
     d. If the value of lump sum tax is Rs.12,920 what would be the profit of the firm.   (5 marks)

16. Below information are related to firm in a competitive market.

     Qs = -a + bp supply function
     Qd = c - dp demand function
     a,b,c,d ≥ 0

     a. Find the equilibrium price and quantity.   (4 marks)
     b. Assume government imposed per unit tax (T). Derive new supply function after tax.   (4 marks)
     c. Find the new equilibrium price and quantity after tax.   (2 marks)
     d. Determine the value of tax (T) that maximizes tax revenue.   (5 marks)

17. Consider the following production function. Firm can buy capita (K) at $ 12 a unit and labo·· at $ 3 a unit.
$$Q = 25K^{0.5}L^{0.5}$$

     a. Assume firm decides to produce 1250 units. Find out the number of units of capital and labour that minimise the costs   (4 marks)
     b. Calculate the total minimum production cost.   (4 marks)
     c. Calculate the value of lagrange multiplier.   (2 marks)
     d. What would be the effect on cost if firm decreases output by 1 units. (5 marks)