UNIVERSITY OF COLOMBO - SRI LANKA

FACULTY OF ARTS

FIRST YEAR EXAMINATION IN ARTS

SECOND SEMESTER END EXAMINATION - 2023/2024

FND 1206 – INTERMEDIATE MATHEMATICS

ANSWER ANY FOUR (04) QUESTIONS

TWO (02) HOURS

Calculators are not allowed
Graph papers are provided

01.

i. Assuming that $\frac{2x^5 - x^{3/2} + 5x^{3/2}}{\sqrt{x}}$ can be written in the form of $2x^p(x^q + 2)$.

Find the values of p and q. (05 Marks)

ii. A rectangular room length is $2 + 3\sqrt{7}\,m$ and width is $2\sqrt{5} + 3\sqrt{7}\,m$. Find the area of the room. (05 Marks)

iii. Prove that $\log_a 1 = 0$ using indices rule. (05Marks)

iv. Given that $y = 3x^2$,

a. Show that $\log_3 y = 1 + 2\log_3 x$ (05 Marks)

b. Hence, or otherwise solve the equation

$1 + 2\log_3 x = \log_3(28x - 9)$ (05 Marks)

02.

a.

i. Write down the formulas for both simple interest rate and compound interest rate, and define all the variables used in each formula. (05 Marks)

ii. First National Bank is offering 4.25% interest rate on an account. Susan makes an initial deposit of Rs.20,000. Calculate the interest earned over 20 years if the bank calculates the interest using compound interest rate, compounded monthly.

(05 Marks)

b.

i. Kumari is planning to buy a new textbook. She has saved Rs.650 and plans to spend no more than Rs.800. The price of the textbook she wants is Rs.x. She also needs to pay an additional Rs.50 for a textbook cover and a Rs.25 tax fee. Write an inequality that represents the total cost Kumari can afford, and solve for the maximum price of the textbook she can buy. (05 Marks)

ii. The sum of Sarini's age and Hasini's age is 102 years. The difference between Sarini's age and Hasini's age is 52 years. Sarini is older than Hasini. Find the age of each woman by using simultaneous equations. (05 Marks)

iii. Use long division to simplify the following expression:

$$(5x^4 - 3x^3 + 2x^2 - 1) \div (x^2 + 4)$$ (05 Marks)

03.

i. Let, $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Write a general formula for the inverse of matrix M, in terms of a, b, c, and d. (02 Marks)

ii. Hence find the inverse of matrix B = $\begin{pmatrix} 5 & -8 \\ -2 & 5 \end{pmatrix}$ if it exists. (03Marks)

iii. If $\begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix} + \frac{1}{2}B = -1\begin{pmatrix} 9 & -3 \\ 11 & -12 \end{pmatrix}$ Find the matrix B. (04 Marks)

iv. If $A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & 2 \\ -1 & 0 & -1 \\ -3 & -3 & -2 \end{pmatrix}$

Given that, $\{kA + (1-k)B\}^2 = I$; where k is a constant that is not equal to zero. Find the value of k. (08 Marks)

v. Solve the following system of equation using Cramer's Rule

$$4x + 3y - 2z = 7$$
$$x + y = 5$$
$$3x + z = 4$$ (08 Marks)

04.

a.

i. Solve the following linear equation.

$$\frac{2x-1}{3}-\frac{3x}{4}=\frac{5}{6}$$

(03 Marks)

ii. Find the equation of the line in the form $y = mx + c$ and plot the graph of the line. The line passes through the points $(-1,3)$ and $(2,7)$.

(04 Marks)

b. A function is given by:

$$y-(5x-3)^2+2(x^2-2)^2=-31x^2+29x+2+2(x^4-3x)$$

i. Simplify and factorize the above expression completely. Write the final answer in the form:

$$y=(ax-b)(x-c);$$

Where a, b and c are constant to be determined. (08 Marks)

ii. Hence, find the roots of the equation $y = 0$. (02 Marks)

iii. Sketch the graph of y, clearly showing the x-intercepts, y-intercept, and the axis of symmetry. (08 Marks)

05.

i. Susantha has some flowers in a bag. 5 of the flowers are orange. 3 of the flowers are purple. The rest of the flowers are pink. Susantha takes a flower from the bag at random. The probability that she takes a pink flower is $\frac{3}{5}$. How many pink flowers are in the bag before Susantha takes a flower?

(05 Marks)

ii. In a group of 150 students, the following information is provided:100 students are enrolled in Mathematics (denoted by M). 60 students are enrolled in Statistics (denoted by S). All students who are enrolled in Statistics are also enrolled in Mathematics. Draw a Venn diagram to represent the situation described.

(05 Marks)

iii. In a university, there are 100 students in the Sinhala department. 60 are females and 40 are males. Among the students, 25 females and 15 males received an A grade, 20 females and 10 males received a B grade, and 10 females and 5 males received a C grade. If one student is chosen randomly from the class.

a. Find the probability that the student is female and received an A grade.

b. Find the probability that the student is male or received a B grade.

c. Find the probability that the student is female and did not receive a C grade.

(15 Marks)

06.

i. State the general formula for differentiating the product of two functions $U(x)$ and $V(x)$, and for the quotient of two functions $\frac{U(x)}{V(x)}$ where $U(x)$ and $V(x)$ are differentiable functions.

(02 Marks)

ii. Find the differentiation of $y = \frac{2x^4}{(x+1)(x^2+2)}$ (08 Marks)

iii. Find the differentiation of $y = ln\left(\frac{2x^4}{(x+1)(x^2+2)}\right)$ (05 Marks)

iv. Consider following equation,

$$y = -x^2 + 2x + 3$$

Find the turning points of the curve and determine the turning point is maximum or a minimum. (05 Marks)

v. Hence, sketch the curve, indicating the location of the turning points. (05 Marks)

07.

i. Factorize the following function.

$$81a^2 - 121b^2c^2$$ (05 Marks)

ii. Find determinant of following matrix V

$$V = \begin{pmatrix} 1 & -3 & 2 \\ 10 & 4 & 0 \\ -1 & 0 & 3 \end{pmatrix}$$ (05 Marke)

iii. A bag contains 3 red balls and 2 green balls. A ball is selected at random, and then another ball is selected without replacement. Draw the tree diagram to represent this scenario. (05 Marks)

iv. Get the first and second derivative of the following function.

$$y = \frac{1}{2}x^2 - \frac{1}{x^7} + 3\sqrt[3]{x} + 3$$ (05 Marks)

v. Integrate the following function.

$$\int (8x^3 - \frac{3}{2\sqrt{x}} + 5)\, dx$$ (05 Marks)