UNIVERSITY OF COLOMBO - SRI LANKA

FACULTY OF ARTS

FIRST YEAR EXAMINATION IN ARTS

FIRST SEMESTER END EXAMINATION - 2023/2024

FND 1106 – INTERMEDIATE MATHEMATICS

Two (02) Hours

INSTRUCTIONS:

The question paper is comprised of **PART A** and **PART B**

PART A has **SIX (06)** questions (I-VI). Answer any **FOUR (04)** questions.

PART B has **FIVE (05)** questions (1-5). Answer any **THREE (03)** questions.

Calculators are not permitted.

Read the instructions carefully.

Provide workings for each answer.

Graph papers are provided

PART A - 40 marks – Answer any **Four (04)** questions. Each question carries **TEN (10)** marks equally divided among parts thereof.

I. Determine the value of the following:

a. $\dfrac{32}{\sqrt[3]{\dfrac{(-128)}{-2}}}$

b. $\left(-\frac{125}{27}\right)^{\frac{1}{3}}\left(\frac{1}{3}\right)^{-2}$

c. $\sqrt[3]{\left(\frac{8}{27}\right)^2}$

d. $\dfrac{\sqrt[3]{-500}}{\sqrt[3]{2}}$

II. Simplify the following.

a. $-\frac{1}{3}(6x-1)+\frac{1}{2}(4y-1)-(-2x+3)$

b. $t\{2-5x(t+2)(t-1)+5[2t(x-3)]\}$

c. $\left(\sqrt{t^2-1}-5\sqrt{3}\right)\left(5\sqrt{3}+\sqrt{t^2-1}\right)$

d. $-5(4\alpha-2\beta+1)+10(\alpha-3\beta+2)$

III. Factorize following functions.

a. $(t-6)(t+9)^3+(t+9)^2(t-6)^2$

b. $54x^3y+81x^4y^2$

c. t^2-t^3+t-1

d. $25(a+b)^2-(a-b)^2$

IV. Complete as per the instructions

a. Subject the "n" in given expression.

$$S=P(a+nt^2)^{\frac{1}{7}}$$

b. Subject the "y" in given expression.

$$\frac{y-a}{b-y}=\frac{y-b}{a-y}$$

c. Simplify, $\sqrt[3]{250}+2\sqrt{98}+\sqrt[3]{54}-\sqrt{50}$

d. Simplify, $\left(\frac{4x^2y+12xy+9y}{xy-2x+3y-6}\right)\div\left(\frac{2x^3y+3x^2y}{xy-2x+3y-6}\right)$

V. Rationalize the denominator and simplify the expression as much as possible.

a. $\frac{8}{7\sqrt{3}}$

b. $\frac{2\sqrt{7}}{\sqrt{7}-\sqrt{2}}$

c. $\frac{\sqrt{3}}{\sqrt{x}-\sqrt{2}}$

d. $\frac{2xy}{\sqrt{8}}$

VI. Complete as per Instructions.

a. Solve for x : $\sqrt{3x-2}-\sqrt{3x}=2$

b. If $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=2\sqrt{3}$, Find $\left(x+\frac{1}{x}\right)$.

c. Solve for t: $\frac{5t+2}{3}-\frac{2t+3}{9}=-t$

d. Divide Rs.700 in to ratio 5: 3 : 2

PART B – 60 marks

Answer any **THREE (03)** questions

Each question carries **TWENTY (20)** marks

1. Answer the following logarithm questions. **[20 marks]**

   a. Evaluate each of the following expressions without using log tables.

      i. $\log_7 \sqrt[3]{49} + \log_7 7 + \log_7 \sqrt[4]{7^2}$ (3 marks)

      ii. $\log_5 \frac{20}{6} + \log_5 3 - \log_5 \frac{2}{125}$ (3 marks)

   b. Given that $\log_{10} 3 = 0.4771$ and $\log_{10} 2 = 0.3010$, Evaluate

      i. $\log_{10} 18$ (4 marks)

      ii. $\log_{10} \sqrt{\frac{6}{2}}$ (4 marks)

      iii. $\log_2 100$ (6 marks)

2. Answer the following matrix questions. **[20 marks]**

   a. If the two matrices are equal, find the value of the unknown terms (6 marks)

$$\begin{bmatrix} 7 & 5 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} x-3 & z-5y+2x \\ 3x+y & 3 \end{bmatrix}$$

   b. Let,

$$A = \begin{bmatrix} 5 & 2 & 2 \\ 6 & 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 1 \\ 6 & 2 & 0 \\ 3 & -2 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad and \quad D = \begin{bmatrix} -2 & 5 & 7 \\ 2 & 3 & 0 \end{bmatrix}$$

   Can you define AB, BD and AC? If can find each. (9 marks)

   c. Find the determinant of B. (5 marks)

3. Answer as per instructions in each part: **[20 marks]**

a. Find the first and second derivatives of the following functions.

i. $R = 8t^2 + 5t - 6$ (3 marks)

ii. $C = 4t^{-3} - 9t^2 - \frac{28}{t^7} - \frac{68}{\sqrt[4]{t}}$ (3 marks)

iii. $y = x$ (3 marks)

iv. $y = x^2 - \sqrt[3]{x} - \frac{1}{x} + 5$ (3 marks)

b. Differentiate each of the following functions using product rule and quotient rule.

i. $Z = (4x^2 - 3)2x^5$ (4 marks)

ii. $y = \frac{5x^2 - 9x + 8}{x^2 + 1}$ (4 marks)

4. Answer as per instructions in each part: **[20 marks]**

a. Consider the equation $y = -\frac{1}{2}(x - 1)^2 + 2$. Answer the following questions.

i. Determine whether the Parabola opens upward or downward, and provide the reason. (01 mark)

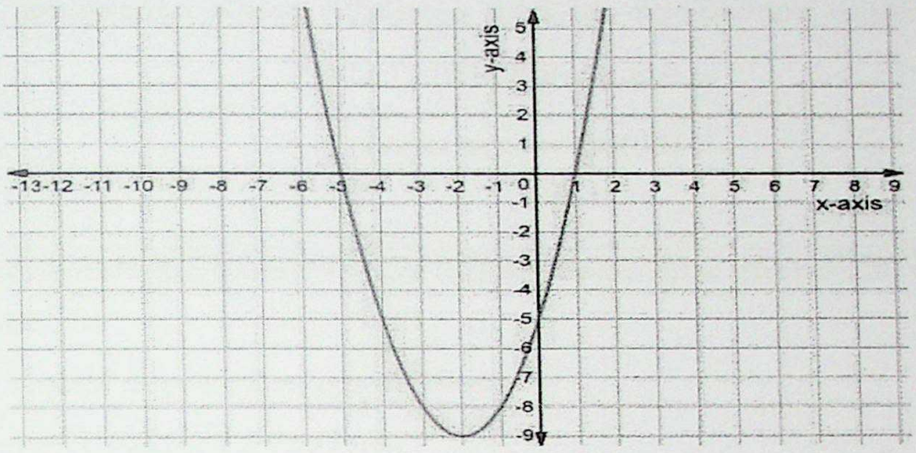
ii. Find the y intercept. (01 mark)

iii. Find the intersection of the graph with the x-axis. (02 marks)

iv. Find the vertex coordinates. (03 marks)

v. Sketch the graph. (05 marks)

b. Using the graph, answer the following questions.



i. Find the value of y intercept. (01 mark)

ii. Find the intersection/s of the graph with the x-axis. (02 marks)

iii. If the intersections of the graph with the x-axis values are a and b, we can derive this graph equation as $y = (x - a)(x - b)$. Then find the real equation for above graph using part (ii) values. (05 marks)

5. Answer as per instructions in each part: **[20 marks]**

a. Solve the following simultaneous equation system.

$$x - 2y + 3z = 9$$
$$-x + 3y - z = -6$$
$$2x - 5y + 5z = 17$$

(12 marks)

b. Solve $2x^3 - 3x^2 + 4x + 5 \div (x + 2)$ (8 marks)

---------- End of the Paper ----------