UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF TECHNOLOGY

LEVEL II EXAMINATION IN TECHNOLOGY - SEMESTER I - 2021

IC 2202 – Discrete Mathematics

Two (02) hours

Answer all the questions

No. of pages: 05

Important Instructions to Candidates

- If a page or part of this question paper is not printed, please inform the supervisor immediately
- Enter your index number on all pages of the answer script
- Write the answers to the questions in the space provided in the question paper.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.

Index No: ……………...

Answer all the questions

1) a. Find the validity of the following arguments. (6 marks)

i. Each autumn either I visit my family, or I take a road trip and take off some time from work. I visit my family only if I do not take off time from work. Therefore, if I take off time from work, then I take a road trip.

ii. If the prime interest rate goes up, then unemployment goes down and prices go up. However, unemployment goes down only if the prime interest rate goes up. Therefore, if prices do not go up, then unemployment does not go down.

b. Write the following statements in propositional logic. Are there logically equivalent pairs of statements? (6 marks)

i. If Mary likes hockey, then she likes swimming and skating.

ii. If Mary likes skating, then she likes swimming and hockey.

iii. If Mary dislikes skating or dislikes swimming, then she dislikes hockey.

c. Let $\mathbb{Z}$ indicate the set of all integers and $\mathbb{R}$ the set of real numbers. (9 marks)
Which of the following quantified predicates are true? Find counter examples for predicates that are false.
Find the negation of these quantified predicates.

i. $\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}, x + 3y = 4$

ii. $\exists x > 0, \forall y > 0, x \cdot y < x$

iii. $\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}, x + y = 15$

d. Simplify the following using propositional algebra. Specify all the steps of derivation. (4 marks)

i. $(x \wedge y) \vee (x \wedge \neg y) \vee (\neg x \wedge y) \vee (\neg x \wedge \neg y)$

ii. $(\neg p \vee q) \wedge (p \vee \neg q)$

2) a. Given the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and sets A = {2, 4}, B = {1, 2, 8} and C = {1, 2, 5, 6, 10}, find each of the following. (8 marks)

i. A x B

ii. P(B) - P(B ∩ C)

iii. (A x C) – (A x A)

iv. P(A ∪ B ∪ C)

b. Determine the size of the following sets. Specify all the intermediate steps. (6 marks)

i. $\{S \in P(\{1,2,3\}) : n(S) \geq 2\}$

ii. $\{S \in P(\{1,2,3\}) : S \cap \{1,2\} \neq \emptyset\}$

c. Identify whether the following definitions represent a function. If it is a function, determine whether it is injective, surjective, or bijective? (6 marks)

i. $h: N^3 \rightarrow N$ where h is given by $h(x, y, z) = x + y - z$

ii. $g: \mathbb{R} \rightarrow \mathbb{R}$ where g is defined by,

$$g(x) = \frac{1}{\sqrt{x+1}}$$

iii. $f: z^2 \rightarrow N$ where f is defined by $f(x, y) = x^2 + 2y^2$

d. Consider a function g which is an invertible function from B to C and h which is also an invertible function from A to B. Prove that the inverse of $(g \circ h)$ is given by $(g \circ h)^{-1} = h^{-1} \circ g^{-1}$ (5 marks)

3) a. Suppose a function f has the domain {a, b, c, d}, codomain {1, 2, 3, 4} and rule {(a,2), (b,1), (c,4), (d,3)}. The function g has the domain {1,2,3,4}, codomain {p, q, r} and rule {(1, p), (2, q), (3, r), (4, p)}. (5 marks)
Find $g \circ f$.

b. Which of the following relations are symmetric, antisymmetric or neither? Support your answer with relevant examples. (9 marks)

i. The relation R on set A = {2, 3, 5, 7} is defined by the rule,
R = {(x, y) ∈ A x A : $x + y$ is even}

ii. The relation R on the set Z is defined by the rule,
R = {(x, y) ∈ ℤ x ℤ : $xy + y$ is even}

iii. R = {(x, y) ∈ ℤ x ℤ : $x^2 + y$ is odd }

c. A relation can be denoted by xRy, where x is an element from set X and y is an element from set Y. (6 marks)

Show that the relation R on ℤ × ℤ defined by, (p, q)R(r, s) if and only if

p + s = q + r is an equivalence relation.

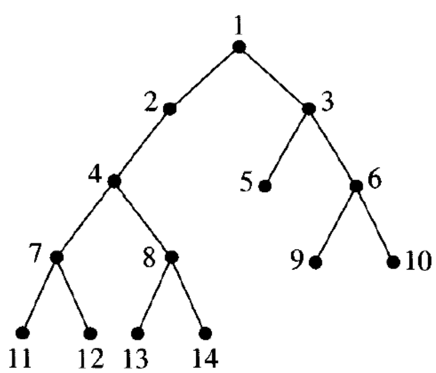
d. Suppose there are two relations defined on set A as R and S. If both the relations are reflexive, what can you say about the reflexive relation on R∩S and R∪S? (5 marks)

4) a. For the relations R_1 and R_2 defined on A = {1, 2, 3 ,4}, construct the graph and the adjacency matrix. (8 marks)

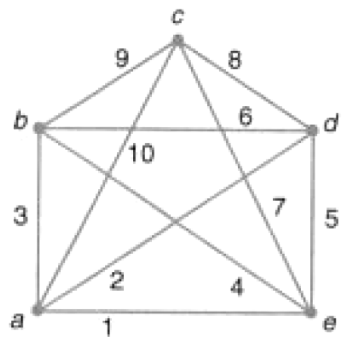
R_1 = {(1,1), (2,3), (3,2), (4,1), (4,4)}

R_2 = {(1,1), (2,2), (3,3), (4,4), (1,3), (3,1), (1,4), (2,4)}

b. Write the postorder traversal of the following tree. (7 marks)



c. Find the minimum spanning trees of the graph using the Kruskal's method. What is the minimum weight? Explain the important steps used to derive your answer. (10 marks)



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