UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF TECHNOLOGY

LEVEL II EXAMINATION IN TECHNOLOGY - SEMESTER II - 2019

IA 2016 – ORDINARY DIFFERNCIAL EQUATIONS

Two (02) hours

Answer all the questions.

Electronic calculators are not allowed.

No. of pages: 09

Important Instructions to Candidates

- If a page or part of this question paper is not printed, please inform the supervisor immediately.
- Enter your index number on all pages of the answer script.
- Write the answers to the questions in the space provided in the question paper.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.

Index No: ……...

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PART A

Write your answers within the given space. You may request additional papers if needed.

1.

a) Show that the equation $P(x,y)dx+Q(x,y)dy=0$ is exact, if $\frac{\partial P(x,y)}{\partial y}=\frac{\partial Q(x,y)}{\partial x}$

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(7 marks)

b) Solve the equation $\left(y^2+x^2-2x+3\right)dx+\left(2xy-y^2+10\right)dy=0$.

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(9 marks)

c) Show that the equation $\left[2\left(x+y^2\right)\cos x+\sin x\right]dx+2y\sin x\,dy=0$ has an integrating factor which only depends on x and **hence** find its solution.

(9 marks)

2.

a) Convert Bernoulli's Equation $\frac{dy}{dx}+p(x)y=q(x)y^2$ to the general form of the first order linear equation, where $p(x)$ and $q(x)$ are independent of y.

(8 marks)

b) Find the general solution of $\frac{dy}{dx} - \frac{y}{x} = xy^2$.

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(8 marks)

c) Reduce the following differential equation to the separable form and find the solution.

$y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

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(9 marks)

3.

a) By the elimination of constants A and B, find the differential equation of which $y = e^x(A\cos x + B\sin x)$ is a solution.

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(7 marks)

b) Reduce the equation given below into the homogeneous form.

$(2x + y - 3)dy - (x + 2y - 3)dx = 0$

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(4 marks)

c) **Hence,** find the solution of the differential equation obtained in Question 3 (a) using a suitable method.

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(7 marks)

d) If $f(x,y)=\tan^{-1}\left(\frac{x}{y}\right)$ show that $\frac{\partial^2 f}{\partial x \partial y}=\frac{\partial^2 f}{\partial y \partial x}$.

(7 marks)

PART B

This part consists of fifteen multiple choice questions. Each question is followed by four choices. Read each question carefully and underline the CORRECT answer. There is only one CORRECT answer for each question.

1) The ordinary differential equation $2x^2\frac{dy}{dx}=x^2+y^2$ can be transformed into the separable form of,

a) $2x\frac{dv}{dx}=v^2-2v+1$
b) $2x\frac{dv}{dx}=\frac{1}{2}v^2-2v+1$
c) $2x\frac{dv}{dx}=v^2+2v+1$
d) $2x\frac{dv}{dx}=v^2+2v-1$

2) All the singular points of the differential equation $\left(y^2-1\right)\frac{dy}{dx}=x\left(x^2+1\right)$ are

a) (0, 1), (0, -1)
b) (1, -1), (1, 0)
c) (-1, 0), (1, -1)
d) (1, 1), (-1, -1)

3) An integrating factor of the differential equation $x^2\frac{dy}{dx}-2xy=4x^4-3x^2y$ is

a) $\frac{1}{x^2}$
b) e^{2x}
c) e^{-2x}
d) None of the above

4) If $y=e^{4x}\sin 3x$, what is the value of $\frac{dy}{dx}$, when $x=\frac{\pi}{3}$?

a) $\frac{dy}{dx}=3e^{\frac{4\pi}{3}}$
b) $\frac{dy}{dx}=0$
c) $\frac{dy}{dx}=-3e^{\frac{4\pi}{3}}$
d) $\frac{dy}{dx}=-e^{\frac{4\pi}{3}}$

5) $Y = Ae^{\left(x+\frac{x^2}{2}\right)} - 1$, where A is a constant, is a solution of the differential equation of,

a) $Y' = (1+y)(1+x)$
b) $Y' = (1-y)(1+x)$
c) $Y' = (1+y)(1-x)$
d) $Y' = (1-y)(1-x)$

6) What is the differential equation which gives the solution of $y^2 = 4a(x+a)$?

a) $y^2\left(\frac{dy}{dx}\right) + 2xy\left(\frac{dy}{dx}\right)^2 - y^2 = 0$
b) $y^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} - y^2 = 0$
c) $y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right)^2 - y^2 = 0$
d) None of the above.

7) The differential equation $P(x,y)dx + Q(x,y)dy = 0$ is said to be exact if,

a) $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$
b) $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$
c) There exists u such that $\frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial x \partial y}$
d) There exists u such that $\frac{\partial Q}{\partial y} = \frac{\partial^2 u}{\partial y \partial x}$

8) What is the first derivative of the equation of $y = e^{k\sin^{-1}x}$?, where k is a constant.

a) $\frac{dy}{dx} = \frac{ky}{\sin^{-1}x}$
b) $\frac{dy}{dx} = \frac{ky}{\sqrt{1-x^2}}$
c) $\frac{dy}{dx} = \frac{ky}{\sqrt{\sin^{-1}x}}$
d) $\frac{dy}{dx} = -\frac{k}{\sqrt{1-x^2}}$

9) The partial fraction of $\frac{5x^2+17x+15}{(x+2)^2(x+1)}$ is,

a) $\frac{2}{x+2}+\frac{1}{(x+2)^2}+\frac{3}{x+1}$

b) $\frac{2}{x+2}-\frac{1}{(x+2)^2}+\frac{3}{x+1}$

c) $\frac{2}{x+2}-\frac{1}{(x+2)^2}-\frac{3}{x+1}$

d) $\frac{3}{x+2}+\frac{1}{(x+2)^2}+\frac{2}{x+1}$

10) Find the solution of $\int \frac{1}{2\sqrt{4-x^2}}$.

a) $\frac{1}{4}\sin^{-1}\left(\frac{x}{2}\right)+C$

b) $\frac{1}{4}\sin^{-1}(x)+C$

c) $\frac{1}{8}\sin^{-1}\left(\frac{x}{2}\right)+C$

d) $\frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right)+C$

Where C is an arbitrary constant.

11) The order and the degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=\frac{d^2y}{dx^2}$ respectively are,

a) 2 and 1
b) 1 and 3
c) 3 and 2
d) 2 and 2

12) Solution of the differential equation $\frac{dy}{dx}=\frac{x^2-1}{y+5}$ is

a) $\frac{x^3}{3}+x-\frac{y^2}{2}-5y=C$

b) $\frac{x^3}{6}-x+\frac{y^2}{2}-5y=C$

c) $\frac{x^3}{3}-x-\frac{y^2}{2}-5y=C$

d) $\frac{x^3}{6}+x+\frac{y^2}{2}+5y=C$,

Where C is an arbitrary constant.

13) Which of the following differential equation is exact?

a) $x\sin y dy + \frac{1}{2}x^2 \cos y dx = 0$

b) $\frac{xy-1}{x^2 y}dx - \frac{1}{xy^2}dy = 0$

c) $\left(5x^2 y - 3x^2 y^3\right)dx + \left(3xy^2 - 5x^2 y^2\right)dy = 0$

d) $\left(3x^4 y^2 - x^2\right)dy + \left(4x^3 y^3 + 2xy\right)dx = 0$

14) The solution of the differential equation $\frac{dy}{dx} - x\tan\left(y-x\right) = 1$ is,

***Hint*:** Take $t = y - x$

a) $\ln\sin\left(y-x\right) = x + C$

b) $\ln\sin\left(y-x\right) = \frac{x^2}{2} + C$

c) $-\ln\cos\left(y-x\right) = \frac{x^2}{2} + C$

d) $\ln\tan\left(y-x\right) = \frac{x^2}{2} + C$

Where C is an arbitrary constant.

15) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + x\sin\frac{y}{x}$ is,

Hint: $\left(\int \frac{dx}{\sin x} = \ln\tan\frac{x}{2}\right)$

a) $\ln\tan\frac{y}{2x} = x + C$

b) $\ln\tan y = x + C$

c) $\ln\tan x = x + C$

d) $\ln\tan\frac{y}{2x} = C$

Where C is an arbitrary constant.

(25 marks)