UNIVERSITY OF COLOMBO, SRI LANKA
FACULTY OF TECHNOLOGY
LEVEL I EXAMINATION IN TECHNOLOGY – Semester II – 2019

IA 1005 – Vectors and Matrices

Answer **4 questions out of 6**

Time: **Two (02) hours**

No. of Pages: **6**

Important Instructions to Candidates

- If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- Enter your Index Number on all pages in the answers script.
- **STRUCTURED ESSAY TYPE: Write the answers to the questions on booklets provided.**
- ESSAY TYPE: Write the answers to the questions on booklets provided.
- Calculators are **NOT** allowed
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.

1 (25 marks)

A Let the matrix A=$\frac{1}{2}\begin{bmatrix}1+i & -1+i\\ 1+i & 1-i\end{bmatrix}$

i Find the conjugate of matrix A (2 marks)

ii Hence show that A is a Unitary matrix (6 marks)

B Let A =$\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$ and B = =$\begin{bmatrix}1 & -2\\ -1 & 0\\ 2 & -1\end{bmatrix}$

Obtain the product of AB and BA (6 marks)

C

i Evaluate the determinant of the matrix A =$\begin{bmatrix}2 & 3 & 5\\ 4 & 1 & 0\\ 6 & 2 & 7\end{bmatrix}$ (5 marks)

ii Find the value of x, if the area of the triangle is 35cm^2 with (x,4), (2,-6), (5,4) using matrix method (6 marks)

2 (25 marks)

A Consider the following system of Equations

$$x + y + z = -1$$

$$x + 2y = 3z = -4$$

$$x + 3y + 4z = -6$$

i Show that the given set of equations are consistent with unique solution (3 marks)

ii Then solve them using the method called Cramer's Rule (12 marks)

B If a matrix A = $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$

i Find the transpose of A (2 marks)

ii Find A^{-1} (6 marks)

iii Hence Prove that $A^T = A^{-1}$ (2 marks)

3 (25 marks)

If matrix A = $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

i Find Eigen values of A (6 marks)

ii If matrix A is diagonalisable, **write** the diagonal matrix of A using the Eigen values stated above (2 marks)

iii Find the Eigen vectors corresponds to each Eigen value (6 marks)

iv Obtain matrix P such that $P^{-1}AP$ is a diagonal matrix (5 marks)

v Find the Eigen values of $A^4 - 6A + 3I$ (2 marks)

4 (25 marks)

A Given the two vectors $\vec{a}$ = (1, 1, 1) and $\vec{b}$ = (0, 2, 0)

i Evaluate $\vec{a} + \vec{b}$ (1 marks)

ii Evaluate $\vec{a} \cdot \vec{b}$ (2 marks)

iii Evaluate $\vec{a} \times \vec{b}$ (2 marks)

iv Evaluate $|\vec{b} \times \vec{a}|$ (2 marks)

B ABC is a triangle, and P, Q, R are the mid-points of the respective sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$. Prove that the medians $\overline{AP}$, $\overline{BQ}$, $\overline{CR}$ meet at a single point G which is the centroid. (5 marks)

C Determine λ such that (5 marks)

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{b} = 2\hat{\imath} - 4\hat{k}$

$\vec{c} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$

are coplanar

D Find the volume of parallelepiped if (4 marks)

$\vec{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$, $\vec{b} = -3\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$, $\vec{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$ are the 3 co-terminus edges of the parallelepiped

E Prove that (4 marks)

$\vec{a}$ x ($\vec{b}$ x $\vec{c}$) + $\vec{b}$ x ($\vec{c}$ x $\vec{a}$) + $\vec{c}$ x ($\vec{a}$ x $\vec{b}$) =0

5 (25 marks)

A There is a scalar function ϕ (x,y,z) = $xy^2 + yz^3$

i Find the directional derivative of above function in terms of x, y, z (5 marks)

ii Evaluate the directional derivative at the point (1,-1,1) (2 marks)

iii Calculate the directional derivative in the direction of (3,1,-1) (3 marks)

B Find the unit normal to the vector to the surface $x^2 + y^2 + z^2 = 5$ at the point (0,1,2) (5 marks)

C There is a vector field V=$(x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath} + (y^2 - xy)\hat{k}$

i Find the divergence of V (5 marks)

ii Find the curl of V (5 marks)

6 (25 marks)

A Let ϕ be a scalar function where $\phi = \log(x^2 + y^2 + z^2)$. Find grad ϕ and evaluate it at (1,-2,1) (5 marks)

B When a force $F = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ moves a particle along the parabola $y^2 = x$

i Find the work done in terms of x and y (5 marks)

ii If particle moves from origin to (1,1),evaluate the work done by the particle (5 marks)

C The acceleration of the particle at time t is given by

$a = 4cos2t\hat{\imath} - 9sin3t\hat{\jmath} + 6t\hat{k}$

if the velocity v and displacement r be zero at t=0

i Find velocity at any point t (5 marks)

ii Find displacement at any point t (5 marks)