



UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF TECHNOLOGY

LEVEL I EXAMINATION IN TECHNOLOGY - SEMESTER II - 2019

IA 1005 – VECTORS AND MATRICES

Two (02) hours

Answer three (03) of the four questions in PART A and answering PART B is compulsory.

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Electronic calculators are not allowed.

No. of pages: 18

Important Instructions to Candidates

- If a page or part of this question paper is not printed, please inform the supervisor immediately.
- Enter your index number on all pages of the answer script.
- Write the answers to the questions in the space provided in the question paper.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.

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[6 marks]

(iii) Let $A = \begin{bmatrix} \alpha & 1 \\ 1 & 1 \end{bmatrix}$. For which values of α , $\det (A^5 - A^4) = -16$?

(Hint: Use properties of determinants)

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[4 marks]

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[4 marks]

2. (i) Consider the following system of equations;

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8$$

$$6x + y - 3z = 5$$

(a) Show that the given set of equations are consistent with a unique solution.

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[4 marks]

3. (i) If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 7\hat{j} + 5\hat{k}$, and $\vec{c} = -3\hat{i} + 2\hat{j} + 5\hat{k}$, determine

(a) $(\vec{a} \cdot \vec{b})$

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[4 marks]

(b) $(\vec{a} \times \vec{c})$

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[4 marks]

(ii) Find the volume of parallelepiped if,

$$\vec{a} = -2\hat{i} + 7\hat{j} + 4\hat{k}$$

$$\vec{b} = -4\hat{i} + 7\hat{j} - 4\hat{k}$$

$$\vec{c} = 7\hat{i} - 2\hat{j} - 4\hat{k}$$

are the three co-terminus edges of the parallelepiped.

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[5 marks]

- (iv) Find the angle between the surfaces of $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point of $(2, -1, 2)$.

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[5 marks]

4. (i) (a) If $\phi(x, y, z) = 3z^2y - y^3z^2$, find the directional derivative at the point of (1,-2,1).

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[4 marks]

- (b) Find the unit vector normal to the surface $x^2y + 2xz = 4$ at point (2, -2, 3).

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[4 marks]

- (ii) Given the vector field, $\vec{V} = (x^2 - y^2 + 2xz)\hat{i} + (xz - xy + yz)\hat{j} + (z^2 + x^2)\hat{k}$.

- (a) Find curl \vec{V} .

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[5 marks]

- (b) Show that the vector given by $\text{curl } \vec{V}$ at points (1, 2, -3) and (2, 3, 12) are orthogonal.

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[6 marks]

(iii) The acceleration of a particle at time t is given by;

$$\vec{a} = (18 \cos 3t \hat{i} - 8 \sin 2t \hat{j} + 6t \hat{k})$$

If the velocity \vec{V} and displacement \vec{r} be zero at $t = 0$;

(a) Find its velocity \vec{V} at any point t .

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[3 marks]

(b) Find its displacement \vec{r} at any point t .

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[3 marks]

PART B

Answering **PART B** is compulsory.

This part consists of fifteen multiple choice questions. Each question is followed by four choices, numbered (i) - (iv). Read each question carefully and circle the number that corresponds to the CORRECT answer. There is only one CORRECT answer for each question.

01. Two matrices A and B are multiplied to get AB if,

- (i) both are rectangular
- (ii) both have same order
- (iii) number of columns of A is equal to number of columns of B
- (iv) number of rows of A is equal to number of columns of B

02. Transpose of a column matrix is a

- (i) row matrix
- (ii) column matrix
- (iii) zero matrix
- (iv) diagonal matrix

03. Which is the singular matrix among follows?

- (i) $\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$
- (ii) $\begin{bmatrix} 3 & -4 \\ -4 & 8 \end{bmatrix}$
- (iii) $\begin{bmatrix} 3 & 2 & 4 \\ 5 & 1 & 6 \\ 2 & 0 & 3 \end{bmatrix}$
- (iv) $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 7 & 3 \\ 5 & 8 & 1 \end{bmatrix}$

04. What is the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$?

- (i) 0
- (ii) 1
- (iii) 2
- (iv) 3

05. What are the eigenvalues of $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$?

- (i) -2, 5
- (ii) -1, 5
- (iii) -2, 4
- (iv) -2, 6

06. Suppose $B = (b_{ij})_{3 \times 2}$ where $b_{ij} = i + 2j$. Which of the following is equal to B?

- (i) $\begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 7 & 8 \end{bmatrix}$
- (ii) $\begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \end{bmatrix}$
- (iii) $\begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$
- (iv) $\begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 5 & 7 \end{bmatrix}$

07. The sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are respectively;

- (i) 7 and 7
- (ii) 7 and 5
- (iii) 7 and 6
- (iv) 6 and 5

08. The directional derivative of $\phi = xyz$ at the point (1, 1, 1) in the direction \hat{i} is;

- (i) -1
- (ii) $-1/3$
- (iii) 1
- (iv) $1/3$

09. If $\vec{V} = xy^2\hat{i} + 2yx^2\hat{j} - 3yz^2\hat{k}$, the curl \vec{V} at point (1, -1, 1) is;
- (i) $-(\hat{j} + 2\hat{k})$
 - (ii) $(\hat{i} + 3\hat{k})$
 - (iii) $-(\hat{i} + 2\hat{k})$
 - (iv) $(\hat{j} + 2\hat{k})$
10. If A and B are (3, 4, 5) and (6, 7, 9) respectively, find \overline{AB} .
- (i) $-(3\hat{i} - 3\hat{j} + 4\hat{k})$
 - (ii) $(3\hat{i} + 3\hat{j} - 4\hat{k})$
 - (iii) $-(3\hat{i} + 3\hat{j} + 4\hat{k})$
 - (iv) $(3\hat{i} + 3\hat{j} + 4\hat{k})$
11. What is the area of parallelogram whose adjacent sides are $(\hat{i} + 2\hat{j} - 3\hat{k})$ and $(2\hat{i} - \hat{j} + 4\hat{k})$?
- (i) 5
 - (ii) $5\sqrt{2}$
 - (iii) $5\sqrt{3}$
 - (iv) $5\sqrt{6}$
12. If $A = \begin{bmatrix} -2 & 5 & -3 \\ 4 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 2 & 0 \\ 4 & 5 & 1 \end{bmatrix}$, Which of the following operation is defined?
- (i) $A+2B$
 - (ii) $2C-A$
 - (iii) $B+0_{2 \times 3}$
 - (iv) $C+0_{2 \times 2}$
13. If the system of equations $x + 2y - 3z = 1$, $(\lambda + 3)z = 3$, $(2\lambda + 1)x + z = 0$ is inconsistent, then the value of λ is equal to
- (i) $-1/2$
 - (ii) -3
 - (iii) 2
 - (iv) 0

14. If $A^2 = A$ then matrix A is called
- (i) Idempotent Matrix
 - (ii) Null Matrix
 - (iii) Transpose Matrix
 - (iv) Identity Matrix
15. If $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$ and $|\vec{r}| = r$, then $\text{div } \vec{r}$ is:
- (i) 2
 - (ii) 3
 - (iii) -3
 - (iv) -2

[25 marks]