UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF TECHNOLOGY

LEVEL I EXAMINATION IN TECHNOLOGY - SEMESTER I

IA 1003 – COMPLEX NUMBERS AND COORDINATE GEOMETRY

Two (02) hours

Answer **4 questions out of 5**

No. of pages: 05

Important Instructions to Candidates

- If a page or part of this question paper is not printed, please inform the supervisor immediately
- Enter your index number on all pages of the answer script
- STRUCTURED ESSAY TYPE: Write the answers to the questions on booklets provided.
- Calculators are NOT allowed
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.

1. (25 marks)

a. Find the roots of $x^2 + 6x + 25 = 0$ by using quadratic formula (4 marks)

b. When the given complex number is $z = \frac{1+2i}{1-(1-i)^2}$ find the modulus of z and hence derive the modulus of the conjugate of the z (4 marks)

c. If $z = 3 + 4i$ and $w = 2 - 3i$ find the followings (5 marks)

   a. z1+z2 and show that complex addition is commutative
   b. z1.z2 and keep the answer in $a + ib$ format
   c. Find $|[\frac{z1}{z2}]|$
   d. $|z1.z2|$

d. Find the power of i^{16} and i^{1002} (4 marks)

e. If $z = acis(\frac{-2\pi}{3})$ and $z^{1/2} = 1 - i\sqrt{3}$ find the value a (4 marks)

f. Solve the equation, $z^4 = 16$ using De Moivre's Theorem. Give your answer in Polar form (4 marks)

2. (25 marks)

a. Express the following complex functions in real-imaginary (u+iv) form, given that z=x+iy (5 marks)

   i. $2z^3 - iz^2$
   ii. ze^z

b. Find all the values of the followings (5 marks)

   i. Log $(i^{1/2})$
   ii. $(1 + i)^i$

c. Find which of these are analytic functions (for x and y real) by checking the Cauchy-Riemann relations hold in some neighborhood of the complex plane (5 marks)

   i. $2x + ixy^2$
   ii. $e^y(cosx + isinx)$

(5 marks)

d. Find the derivative of the following complex functions

i. $f(z) = (z^2 - 1)^n$

ii. $f(z) = (z^2 - 1)(3z + 4)$

iii. $f(z) = \frac{z}{(z^3+1)}$

(5 marks)

e. Differentiable? Show that f = $\bar{z}$ is not differentiable

3. (25 marks)

a. Evaluate the following integrals over complex valued function (5 marks)

i. $\int_0^{\pi} e^{it} dt$

ii. $\int_0^1 (t^2 + it) dt$

(5 marks)

b. Evaluate these line integrals along the given path, $\gamma(t)$

i. $\int_{|z|=1} z\, dz \qquad \gamma(t) = e^{it} \quad 0 \le t \le 2\pi$

ii. $\int_{|z|=1} \frac{1}{z^2} dz \qquad \gamma(t) = e^{it} \quad 0 \le t \le 2\pi$

(5 marks)

c. Consider$\sum_{k=0}^{\infty} z^k$. $If\ z = \frac{i}{4}$, expand the series and show the real and imaginary parts of the series separately.

(5 marks)

d. Find the radius of convergence of the following Power Series using the Ratio Test method or by other means.

a. $\sum_{k=0}^{\infty} kz^k$

b. $\sum_{k=0}^{\infty} \frac{z^k}{k!}$

(5 marks)

e. Find the radius of convergence of the $\sum_{k=0}^{\infty} 2^k z^k$using the Root Test method

4. (25 marks)

a. Represent $z1 = 3 + 4i$ and $z2 = 3 - i$ in the same complex plane. Add (5 marks) these two complex numbers using the complex plane and show the steps and the answer of the summation on the same complex plane.

i. Hence Prove that the sum of two complex numbers is represented by the diagonal of a parallelogram

b. From 4 (a), Prove that $|z1 + z2| < |z1| + |z2|$ (5 marks)

i. If z1 = 2 + 3i and z2 = 1 + i, verify that | z1 + z2 | < | z1 | + | z1 |

c. Convert $z = 1 + i\sqrt{3}$ to polar form and $z = 2cis\ \frac{5\pi}{3}$ to rectangular format (5 marks)

d. If $z = 5cis\ (\frac{5\pi}{3})$ and $w = 2cis\ (\frac{\pi}{6})$ find the following by using the properties of complex multiplication and division (5 marks)

i. z.w

ii. $\frac{z}{w}$

e. Find the followings using de Moivre's theorem (5 marks)

iii. $(1 - i)^4$

iv. $(4 - 4\sqrt{3})^{1/3}$

5. (25 marks)

a. Write the theorems for two given lines AB and CD to be parallel and perpendicular on the complex plane. (4 marks)

b. The points O = (0, 0), A=(x, 10) and B=(y, 36) are the vertices of an equilateral triangle on a complex plane. Find the length of AB. (6 marks)

c. Let ABC be a triangle in the complex plane. (3 marks)

i. If D is the midpoint of the BC, find the formula for D in terms of the complex numbers a, b, c (6 marks)

ii. Hence find a formula for centroid of the triangle ABC in terms of the complex numbers a, b, and c.

(6 marks)

d. Let ABC be a triangle inscribe in the complex unit circle, and let $h = a + b + c$. Prove that H is the orthocenter of the triangle ABC