UNIVERSITY OF COLOMBO – SRI LANKA

Faculty of Arts

Second Year Examination in Arts (Economics) -2017

(End of first semester)

ECN 2132 Mathematical Economics

Time Allowed: Two hours (02) only

Answer all questions from section A and 03 questions from section B

Section A

Answer **all questions** in Section A

40 marks for section A.

1. Differentiate the following functions

   i) $y = x^2 - x^{-2} + 20x + 10$

   ii) $y = x^2 + e^{3x^2} + 3x$

2. Differentiate the following functions

   i) $y = (2x + x^{-2})^{15}$

   ii) $y = \frac{3x+2}{x^2+3x+2}$

   iii) $y = (8x^2 + 2)\left(\frac{1}{3}x^3 + 2\right)$

3. Find the following integrals

   i) $\int -20x^{-2} + x + 35\, dx$

   Evaluate the following definite integral

   ii) $\int_1^2 (3x^2 + 3)dx$

4. What is the total differential of following functions

   i) $y = 2x^2 + 0.5z^2 + xz + 20$

   What is the total derivative dy/dx for the following functions?

   ii) $y = 2x^2 + 0.5z^2 + xz + 20$ where $z = 3x^2$

5. ii) Find the multiplication of AB of the below matrices

$$A = \begin{bmatrix} 7 & 8 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 8 & 2 \\ 8 & 6 & 10 \end{bmatrix}$$

Section B

Answer three (03) questions from section B

60 marks for section B. 20 marks for each question.

6. Following information is given related to the firm in a monopoly market.

Total fixed cost (TFC) = 20

Marginal cost (MC) $= 3Q^2 + 4$

Demand equation $40 = 2P + 2Q$

Where Q = Output, P= Price

i) Determine the profit function in terms of Q. (6 Marks)

ii) Find the output level that maximizes the profit. (6 Marks)

iii) Test the second order conditions for profit maximization. (8 Marks)

7. A firm manufactures two goods, X and Y. and sells in two separate markets. The demand functions for two goods and total cost function are given below

$$X = 12 - P_x$$
$$Y = 18 - P_y$$
$$TC(x, y) = x^2 + y^2 + 2xy$$

i) Find the quantities produced of each of X and Y in order to maximize total profit. (6 Marks)

ii) Find the price of each of X and Y in order to maximize total profit. (6 Marks)

iii) Test the second order conditions for profit maximization using Hessian method. (8 Marks)

8. Following production function shows the relationship between output and production factor capital (K) and labour (L).

$$Q = 6K + 0.3K^2L + 1.2L^2$$

Based on the above production function state whether following statements are true or falls and give reasons for your answer.

i) The slope of marginal product of capital curve is increase with L but constant over K. (6 Marks)

ii) For given value of capital marginal product of labour cure is a straight line. (6 Marks)

iii) If $K = 0.8\ L^2$ change of labour has direct and indirect effects on output. (8 Marks)

9. A consumer buys two goods, X and Y, The price of one unit of X is Rs.1.00 and price of one unit of Y is Rs. 16.00. The consumer's utility function is given by

$$U(x, Y) = X^{\frac{3}{4}} Y^{\frac{1}{4}}$$

She has a budget of Rs.1280 to spend on X and Y.

i) Using method of Lagrange multiplier method find the value of X and Y which will maximize the consumer's utility function subject to the given budget constraint. (6 Marks)

ii) What percentage of income will consumer spends on good X at the utility maximization? (6 Marks)

iii) Test the second order conditions for utility maximization using bordered Hessian matrix. (8 Marks)

10. A statistician estimates that a country's population N is growing continuously and can be determined by the following function

$$N = 3{,}620{,}000e^{0.02t}$$

Where t is the number of years after year 2015.

i) What is the population growth rate? (5 Marks)
ii) What is the population of year 2015? (5 Marks)
iii) What will the population in year 2050? (5 Marks)

iv) GDP of a hypothetical country has increased from Rs.10, 361 billons to Rs 10,952 billons in year 2015. If this growth rate continued, what would GDP be at year 2020? (5 Marks)