

UNIVERSITY OF COLOMBO, SRI LANKA
FACULTY OF ARTS
FIRST YEAR EXAMINATION IN ARTS (SEMESTER I) – 2017/2018
FND 1106 – INTERMEDIATE MATHEMATICS
(Time: Two Hours)

Answer any five (05) questions

No. of questions: 07

No. of pages: 05

(Each question carries equal marks)

Calculators are not permitted

01. (a) State whether the following numbers are rational or irrational.

$$x = \frac{\sqrt{150} \sqrt{125}}{\sqrt{2} \sqrt{3} \sqrt{5}} \quad (3 \text{ marks})$$

$$y = \frac{\sqrt{150} \sqrt{125}}{\sqrt{2} \sqrt{3}} \quad (2 \text{ marks})$$

(b) Simplify.

$$a = \frac{\sqrt[4]{81} (7^{-1})^2}{49^{-1}} \quad (5 \text{ marks})$$

$$b = \frac{(2^{-0.5})^4 (2^0)^{2018}}{2^{-1} + 2^{-1}} \quad (5 \text{ marks})$$

(c) Rationalize the denominator and simplify.

$$\frac{13}{4 + \sqrt{3}} \quad (5 \text{ marks})$$

(Total: 20 marks)

02. (a) Solve the following quadratic equations.

(i) $x^2 - 5x + 6 = 0$ (2 marks)

(ii) $x^2 + 2x - 4 = 0$ (3 marks)

(b) Simplify without using log tables.

$$X = \frac{2018}{\lg 100} - \frac{2016}{\log_3 27} \quad (3 \text{ marks})$$

$$Y = \frac{1}{\log_{2000} 10} - \log_{10} 2 \quad (2 \text{ marks})$$

(c) Solve for y .

$$2 \log_a y - \log_a 9 = \log_a 4 \quad (3 \text{ marks})$$

(d) (i) Solve the equation.

$$|7x - 3| = 11 \quad (3 \text{ marks})$$

(ii) Solve the following simultaneous equations.

$$3x - y = 5$$

$$2x + 3y = 18 \quad (4 \text{ marks})$$

(Total: 20 marks)

03. (a) Using the given universal set, $\varepsilon = \{x: 1 \leq x < 20 \text{ and } x \text{ is an integer}\}$

(i) State the following sets by listing elements.

$$P = \{\text{multiples of } 3\}$$

$$Q = \{\text{even numbers}\} \quad (4 \text{ marks})$$

(ii) Find the sets $P \cup Q$, $P \cap Q$, Q^c , $(P \cup Q)^c$. (4 marks)

(iii) Verify that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$ for the sets P and Q . (2 marks)

(b) A jar consists of 21 sweets. 12 are green and 9 are blue. Dimuthu picked two sweets at random. (without replacement)

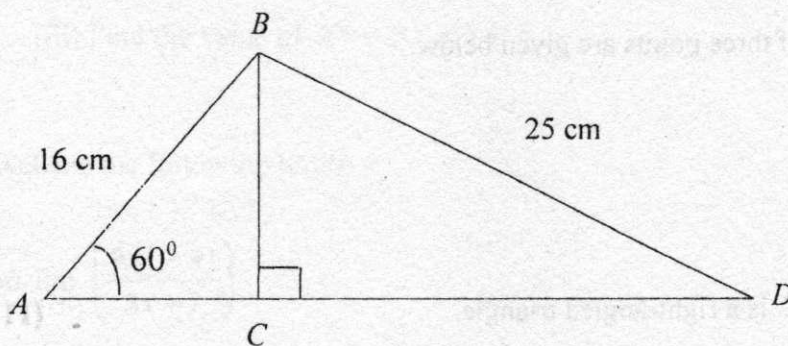
(i) Draw a tree diagram to represent the experiment. (6 marks)

(ii) Find the probability that one sweet is blue and one sweet is green. (2 marks)

(iii) Dimuthu randomly took third sweet. Find the probability that all three sweets are green. (2 marks)

(Total: 20 marks)

04. (a) In the given figure $AB = 16$ cm, $BD = 25$ cm and $\hat{BAC} = 60^\circ$.

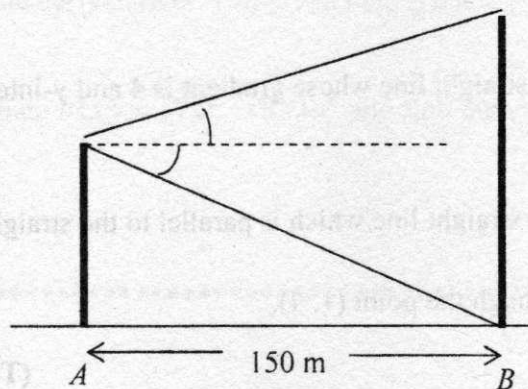


(i) Find the length of AC . (4 marks)

(ii) Find the length of BC . (4 marks)

(iii) Find the value of $\operatorname{cosec} \theta$ if $\hat{BDC} = \theta$. (2 marks)

(b)



The horizontal distance between two towers A and B is 150 m. The angle of elevation of the top of the tower B and the angle of depression of the bottom of the tower B from the top of the tower A are $8^{\circ} 17'$ and $16^{\circ} 23'$ respectively.

(i) Find the height of the tower A . (3 marks)

(ii) Find the height of the tower B . (3 marks)

(iii) Find the angle of elevation of the top of the tower B from the bottom of the tower A . (4 marks)

(Total: 20 marks)

05. (a) Co-ordinates of three points are given below.

$$A = (3, 4)$$

$$B = (3, 1)$$

$$C = (8, 4)$$

Show that ABC is a right-angled triangle. (11 marks)

(b) Show that the mid-point of the line joining the points $(15, 10)$, $(49, 25)$ and the mid-point of the line joining the points $(29, 5)$, $(35, 30)$ have similar co-ordinates.

(4 marks)

(c) The equation of the straight line having gradient m and y-intercept c can be written in the form

$$y = mx + c$$

(i) Find the equation of the straight line whose gradient is 4 and y-intercept is -3.

(2 marks)

(ii) Find the equation of the straight line which is parallel to the straight line

$y = 2x + 1$ and goes through the point $(1, 4)$. (3 marks)

(Total: 20 marks)

06. (a) If $A = \begin{pmatrix} 3 & 5 & 2 \\ 8 & 1 & 4 \\ 1 & 1 & 7 \end{pmatrix}_{3 \times 3}$ and $B = \begin{pmatrix} 1 & -2 & -3 \\ 5 & 0 & -4 \\ 2 & 5 & 8 \end{pmatrix}_{3 \times 3}$, then find $A+B$ and $A-B$.

(6 marks)

(b) $X = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}$, $Y = \begin{pmatrix} 2 & 1 \\ 5 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Find XY .

(2 marks)

(ii) Show that $(XY)^T = Y^T X^T$.

(5 marks)

(iii) Find the value of $X^2 - 3X + 14I$.

(7 marks)

(Total: 20 marks)

07. (i) Evaluate the following limits.

(a) $\lim_{x \rightarrow 1} \left(\frac{9x^2 + 91}{3x + 7} \right)$

(2 marks)

(b) $\lim_{x \rightarrow \infty} \left(\frac{6x^3 + 5x - 3}{3x^3 + 5x^2 - 3} \right)$

(4 marks)

(ii) Let $f(x) = (x^2 + 1)(x^2 + 2)$ and $g(x) = 2x + 7$.

(a) Find the derivative of f and the derivative of g .

(3 marks)

(b) Find the derivative of $f \cdot g$ given that $(fg)' = f'g + g'f$.

(6 marks)

(iii) Differentiate $h(x) = (7x^2 - 3x)e^x$ and find the value of the derivative at $x = 1$.

(5 marks)

(Total: 20 marks)
