UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF ARTS

SECOND YEAR EXAMINATION IN ARTS (ECONOMICS) – 2018/19

ECN 2132 – MATHEMATICS FOR ECONOMICS

Time Allowed: Two (02) Hours

Answer all questions in PART A and 02 questions from PART B.

Calculators can be used.

PART A

(50 marks)

Question (1) is compulsory.

Question (1) **(02 marks for each)**

1. Find the sum of A and B of the following matrices.

$$A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \\ 1 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 0 & 2 \end{bmatrix} \qquad A + B =$$

2. Find the difference between X and Y of the following Matrices,

$$X = \begin{bmatrix} 4 & 16 \\ 10 & 22 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 15 \\ 6 & 3 \end{bmatrix} \qquad X - Y =$$

3. Find the product AB.

$$A = \begin{bmatrix} 3 & 3 \\ 9 & 6 \\ 6 & 5 \end{bmatrix} \qquad B = 4$$

4. $$A = \begin{bmatrix} 8 & 2 & 3 \\ 1 & 9 & 4 \\ 4 & 3 & 6 \end{bmatrix}$$

   Find the determinant of A.

5. Find the first order partial derivative. $Z = 8x^2 + 14xy + 5y^2$

6. Find the second order partial derivative. $Z = x^4 + x^3y^3 - 3xy^3 - 2y^3$

7. Find the cross partial derivative. $Z = 3x^2 + 12xy + 5y^2$

8. Write Young's theorem using an appropriate example.

9. Differentiate with respect to x:

$Y = mx^2 + nx + p$

Where m, n, p are constants.

10. $Q = f(L, t)$ $\qquad$ $L = f(t)$

Find the total derivative of $\frac{dQ}{dL}$ with respect to L.

11. $Z = f(X, Y)$ X and Y are variables.

What are the conditions for maximization of Z.

12. Find the total differential of $U = f(X, Y)$

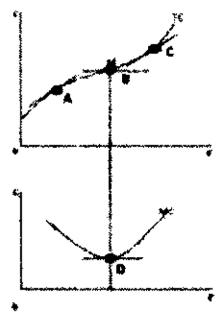
13. Identify the points or/and types of curves marked as follows for values f ′(x) and f ′′(x)

a. A –

b. B –

c. C –

d. D –



14. Population in many third-world countries is growing at 3.2 percent. Calculate the population 20 years from now for a country with 60,000,000 people.

15. Change the logarithm to the equivalent exponential form.

$\log_8 64 = 2$

16. Change the natural logarithm to the equivalent natural exponential form.

$\ln Y = 2t + 1$

17. Current stocks of mineral (M) are 250 million tonnes. If these stocks are continually being used up at an annual rate of 9%, what amount of M will remain after 30 years?

18. Differentiate with respect to x.

$Y = (2x^2 + 3x - 1)(5x^2 - 2x + 3)$

20. Differentiate with respect to x.

$$Y = \frac{2x+5}{3x-1}$$

21. Differentiate with respect to x.

$$Y = \frac{(2x+1)(3x-2)}{(5x+3)}$$

22. Solve. $\int 5\,dx$

23. Solve. $\int dx$

24. Solve. $\int (5x^3 + 2x^2 + 3x)\,dx$

25. Solve. $\int_5^5 (2x+3)x\,dx$

END OF PART A

PAGE TURN OVER FOR PART B

(50 marks)

Answer any 02 questions only.

Question (2) (25 marks)

The following equation shows the relationship between total product and capital of a firm in short run.

$$TP = 90K^2 - K^3$$

i. How many capital units should the firm employ in order to reach maximum total product in the short run? Prove your answer using second order conditions.

ii. Prove that when total product reaches its maximum point, marginal product of capital is equal to zero.

iii. What is the maximum average product of capital in the short run?

iv. Prove that marginal product of capital is equal to the average product of capital at the maximum average product of capital.

v. Sketch the graph of total product of capital, average product of capital and marginal product of capital in a one chart. And identify three stages of production in the short run.

Question (3) (25 marks)

a) The demand and supply function of good X are given below,

$P_d = 42 - 6Q - Q^2$ (Inverse demand function)

$P_s = 6+Q$ (Inverse supply function)

i. Find the equilibrium price and equilibrium quantity of good X.

ii. Calculate consumers' surplus and producers' surplus at the equilibrium point.

b) The Costs curves of good X are given below,

Marginal Cost (MC) = $25 + 30Q - 9Q^2$

Total Fixed Cost (TFC) = 55

Base on above information:

i. Find the total cost function.

ii. Find average cost function.

iii. Find total variable cost function.

Question (4) (25 marks)

The function $U = 2xy$ represent the utility derived by a consumer from the consumption of a certain amount of product X and a certain amount of product y.

Assume price of good X and Y are respectively Rs.3.00 and Rs.4.00. The consumer has a fixed budget of Rs.90.00 to buy good X and Y.

i. Derive the budget constraint.
ii. How many units of good X and good Y, should the consumer purchase to maximize his/her utility?
iii. Calculate maximum utility consumer can gain with the given budget.
iv. Test second order condition for utility maximization using bordered Hessian method.
v. Prove that marginal utility of each rupee spent on good X and Y are equal at the utility maximization point.

Question (5) (25 marks)

A monopolist producing three related goods the demand functions and the cost function are:

$$P_1 = 180 - 3Q_1 - Q_2 - 2Q_3$$
$$P_2 = 200 - Q_1 - 4Q_2$$
$$P_3 = 150 - Q_2 - 3Q_3$$
$$TC = Q_1^2 + Q_1Q_2 + Q_2^2 + Q_2Q_3 + Q_3^2$$

i. Find the total revenue for each good produced by this firm.
ii. Find the profit functions for each good.

Maximize profits for the firm, using

iii. Cramer's rule for the first-order condition
iv. The Hessian for the second-order condition.
v. Find the maximum profits for the goods produced by this particular firm.

END OF PART B

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