

UNIVERSITY OF COLOMBO
FACULTY OF ARTS
SECOND YEAR EXAMINATION IN ARTS (ECONOMICS) -2016/ 2017
END OF THE SECOND SEMESTER
ECN: 2223: QUANTITATIVE TECHNIQUES

TIME ALLOWED: TWO (02) HOURS

Calculators are allowed.

Answer four (04) Questions.

1. Consider the following linear programming problem

$$\text{Max; } Z = 3Y + 3X$$

s.t

$$1X \leq 100$$

$$1Y \leq 80$$

$$2X + 4Y \leq 400$$

$$X, Y \geq 0$$

- i. Draw separate graphs for each constraint and highlight the feasible region. (8 Marks)
- ii. How many extreme points are there? Calculate the value of the objective function at each extreme point? (7 Marks)
- iii. Which extreme point do you consider as an optimal solution? Give reasons for your answer? (5 Marks)
- iv. Assume that the objective function has changed to $2Y + 6X$, What would be the new optimal solution? (5 Marks)

2. Consider the following linear programming problem

$$\text{Mini } C = 30X_1 + 50X_2$$

s.t

$$6X_1 + 2X_2 \leq 30$$

$$3X_1 + 2X_2 \leq 24$$

$$5X_1 + 10X_2 \leq 60$$

- i. Convert above minimization problem in to maximization using dual method. (8 Marks)
- ii. Solve the dual using simplex method. (7 Marks)
- iii. What would be the cost minimizing level of production for X_1 and X_2 ? (5 Marks)
- iv. What would be the minimum cost? (5 Marks)

3. Super Latex Company is introducing two new Sofa sets, the Super Luxury (X1) and Luxury (X2). The anticipated unit profits are Rs.15000 for the Super Luxury and Rs.10000 for the Luxury. Company uses two types of materials to produce sofa. Material requirement per unit of product and available material stocks at the stores are shown in following table.

	<u>Material 1</u>	<u>Material 2</u>
Super Luxury	2	4
Luxury	3	2
Available stocks at stores	100	80

- i. Formulate a linear programming model that can be used to determine the number of super luxury sofa and number of luxury sofa that will maximize total profit. (No need to solve it) (3 Marks)

Following figures shows solver solution and sensitivity analysis report taken from Excel Solver for the above problem. Answer (ii) to (vi) base on the following information

	Super Luxury (X1)	Luxury (X2)	Amount Available
M1	2	3	100
M2	4	2	80
Profit	15000	10000	
Decision Variables			
	Super Luxury (X1)	Luxury (X2)	
Total produce	5	30	
Maximum profit	375000		
M 1	100	<=	100
M 2	80	<=	80

	A	B	C	D	E	F	G	H
1	Microsoft Excel 15.0 Sensitivity Report							
2								
3	Variable Cells							
4				Final	Reduced	Objective	Allowable	Allowable
5	Cell	Name	Value	Cost	Coefficient	Increase	Decrease	
6	\$C\$9	Total produce X1	5	0	15000	5000	8333.333333	
7	\$D\$9	Total produce X2	30	0	10000	12500	2500	
8								
9	Constraints							
10			Final	Shadow	Constraint	Allowable	Allowable	
11	Cell	Name	Value	Price	R.H. Side	Increase	Decrease	
12	\$C\$12	M 1 X1	100	1250	100	20	60	
13	\$C\$13	M 2 X1	80	3125	80	120	13.33333333	

- ii. What would be the profit maximizing production for each sofa type? (2 Marks)
 - iii. What would be the maximum profit that the firm can earn at current situation? (2 Marks)
 - iv. What is the maximum per unit profit that firm can gain from super luxury and luxury sofas without changing the current level of production? (2 Marks)
 - v. Assume due to a change of economic environment, per unit profit from super luxury has increased from Rs. 15000 to Rs. 16500 and per unit profit from luxury has increased from Rs. 10000 to Rs. 12000. What would be the respective levels of production? (6 Marks)
 - vi. Give interpretation for shadow price (dual price) related to each material. (5 Marks)
 - vii. Assume that firm increases material one (M1) by ten units what would be the impact on profit? (5 Marks)
4. A product is produced at three plants and delivered to three markets. The transportation cost per unit and other information are shown in the following table.

Origin	Destinations				Supply
	Market 1	Market 2	Market 3	Market 4	
P1	3	2	7	6	5000
P2	7	5	2	3	6000
P3	2	5	4	5	2500
Demand	6000	4000	2000	1500	

- i. Present the above problem using network representation method. (8 Marks)
- ii. Develop a linear programming model for the above problem. (7 Marks)

The linear model was solved using Excel Solver and results are shown in the following figure. Answer question (iii) to (iv) based on solver results.

	A	B	C	D	E	F	G	H	I	J
1										
2			Destinations							
3		Origin	Market 1	Market 2	Market 3	Market 4	Supply			
4		Plant 1	3	2	7	6	5000			
5		Plant2	7	5	2	3	6000			
6		Plant 3	2	5	4	5	2500			
7		Demand	6000	4000	2000	1500				
8										
9		Model								
10			Min Cost	39500						
11			Destinations							
12		Origin	Market 1	Market 2	Market 3	Market 4	Total			
13		Plant 1	3500	1500	0	0	5000	<=	5000	
14		Plant2	0	2500	2000	1500	6000	<=	6000	
15		Plant 3	2500	0	0	0	2500	<=	2500	
16		Total	6000	4000	2000	1500				
17			=	=	=	=				
18			6000	4000	2000	1500				
19										

iii. What is the total transportation cost related plant 1? (5 Marks)

iv. How does the management plan deliveries to minimize transportation cost? (5 Marks)

5. (a)

The management of DN Company is planning to introduce a new product. The fixed cost for the production is Rs.30,000. The variable cost per unit is expected to be between Rs.16. to Rs.24 and is described by the discrete probability distribution shown in the following table. The product will sell for Rs.50. per unit. The management assumes that the first year demand is described by the normal probability distribution with mean 750 units and standard deviation 250. However, the management believes that in the best case scenario the demand for the product is 1200 units and in the worst case scenario the demand for the product is 500 units.

Probability Distribution for Variable Cost	
Variable Cost	Probability
16.00	0.1
18.00	0.2
20.00	0.4
22.00	0.2
24.00	0.1

- i. Develop a mathematical model for total profit assuming first year demand is Q_x . (2 Marks)
- ii. What is the controllable variable in your model? (2 Marks)
- iii. What are the probabilistic variables in your model? (2 Marks)
- iv. What is the output in your model? (2 Marks)
- v. Identify the parameters in your model. (2 Marks)
- vi. Calculate the profit related to best and worst case scenarios. What are your findings based on your analysis? (5 Marks)

(b) The management has developed a simulation model and the summary results for 100 trials are reported in the following table.

Frequency Distribution	
Profit (Rs.)	Number of Trials
less than or equal to 0	13
1 to 2,499	22
2,500 to 9,999	22
10,000 to 19,999	21
Above 19,999	22
	100

Descriptive Statistics	
Average	8970
Minimum	-12390
Maximum	41881
Standard Deviation	12663

- vii. What is the probability of making losses? (5 Marks)
- viii. What is your advice to the firm base on simulation results? (5 Marks)

6. A garment factory has 4 plants and these plants can produce four major types of garments such as, garment 1, 2, 3 and 4. The optimal allocation of plants will determine the maximum level of efficiency of the respective type of the garment amongst the four main types. The factory manager has got the objective of allocating these plants in such a way that would maximize the total efficiency of the production.

The factory manager estimates that the alternative allocations of the plants for each type of garment would yield the level of efficiency shown in the following table.

Higher values indicate the higher level of efficiency.

Plant	Efficiency levels of the production			
	Type of the garment			
	1	2	3	4
1	3	5	2	6
2	5	5	4	7
3	6	6	7	9
4	7	9	8	9

- i. Graphically illustrate this table using a network representation? (10 Marks)
- ii. Find out the optimal allocation of plants using dynamic programming? (10 Marks)
- iii. What is the optimized level of total production efficiency? (5 Marks)
