UNIVERSITY OF COLOMBO - SRI LANKA

FACULTY OF ARTS

MASTERS IN ECONOMICS -2015/2016

Final Examination -Semester I

MECON 503 : QUANTITATIVE ECONOMICS

Time Allowed: 03 Hours

This paper consists of 8 questions. Answer any five (5) questions. Maximum marks for each attempted question is 20.

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1. (i) Find $\partial f/\partial x$, and $\partial^2 f/\partial y \partial x$ of the following functions.

   (a) $f(x,y) = x^2y + x^2y^2$

   (b) $f(x,y) = 10x + \ln(xy)$

   (c) $f(x,y) = x^3 + 5xy - y^2$

   (ii) The MC of producing a certain good is given by the following function. Determine constants A and B such that

$$MC = \frac{1}{(Q^2+Q-2)} = \frac{1}{(Q-1)(Q+2)} = \frac{A}{(Q-1)} + \frac{B}{(Q+2)}$$

   (iii) Using the above result at (iv) show that total variable cost (VC) producing Q units of the good is given by

$$\ln\left((Q-1)/(Q+2)\right)$$

   (iv) Given the matrices

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1/3 & -1/6 \\ 0 & 1/2 \end{bmatrix}$$

   Verify that AB = BA = I where I is the identity matrix of order 2 x 2.

   (v) Find the ratio of the marginal productivities of labour (MP_L) and capital (MP_K) of the production function given by

$$Q = 8LK - L^2 - K^2,\ L > 0 \text{ and } K > 0$$

2. (i) Consider the following function of two choice variables

$$Z = f(x,y)$$

Write down the first order and the second order conditions for a minimum.

(ii) If $f(x,y) = 3x^2 + y^2 - 3xy$

(a) Find f_x, f_y, f_{xy}, f_{yx}

(b) Show that the function has only one stationary point and classify it.

(c) What is the stationary value of the function?

3. (i) Write the Lagrange function and the first order condition for stationary values (without solving the equations) for the following optimization problem.

Optimize $w = x - y + 2z$ subject to constraints $x^2 + y^2 + z^2 = 1$ and $x + y + z = 0$.

(ii) Use the technique of Lagrange multiplier to find the values of x and y which optimize the function $Z(x,y) = x - 3y - xy$ subject to the constraint $x+y = 6$.

4. (i) Consider the function $z = f(x,y)$ and the total differential of it, dz given by

$$dz = f_x\, dx + f_y\, dy$$

Prove that $d^2z = f_{xx}\, dx^2 + 2 f_{xy}\, dx\, dy + f_{yy}\, dy^2$ where

$f_x = \partial f/\partial x$, $f_y = \partial f/\partial y$, $f_{xx} = \partial^2 f/\partial^2 x$, $f_{xy} = \partial^2 f/\partial x \partial y$, $f_{yy} = \partial^2 f/\partial^2 y$

(ii) Find dz and the d^2z of the function $z = 2x + 9xy - y^2$

5. (i) Under what conditions the following differential equation is exact?

$$M\, dy + N\, dt = 0$$

(ii) Verify each of the following differential equations exact and if so solve them.

(a) $2yt\, dy + y^2\, dt = 0$

(b) $2yt^3 dy + 3y^2t^2\, dt = 0$

6. Briefly explain the meaning of the dynamic stability of a system represented by a differential equation.

   (i) Solve the equation $dy/dt + 4y = 12$, with the initial condition $y(0) = 2$.

   (ii) Discuss the dynamic stability of the solution derived at above (i).

7. (i) Find the inverse of the following matrix

$$B = \begin{bmatrix} 4 & 1 & -1 \\ 0 & 3 & 2 \\ 3 & 0 & 7 \end{bmatrix}$$

   (ii) Using the answer to above (i) solve the following equation system

$$\begin{aligned} 40X + 10Y - 10Z &= 250 \\ 30Y + 20Z &= 300 \\ 30X + 70Z &= 200 \end{aligned}$$

8. A multi - product firm produces two non related products A and B where production functions are given by $Q_A = L^{0.75}K^{0.25}$, $Q_B = L^{0.5}K^{0.7}$. Q_A and Q_B are quantities of outputs of A and B per period of time and L and K are inputs of labour and capital services per period of time. Capital and labour services can be bought at constant prices per unit of Rs. 300 and Rs. 200 respectively.

   (i) Explain the concept of returns to scale in the theory of firm.

   (ii) Derive the returns to scale characteristics of the two production functions.

   (iii) What is the capital to labour ratios for the two plants that minimizes the firms cost at any chosen output levels of two goods?

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