

UNIVERSITY OF COLOMBO, SRI LANKA
FACULTY OF ARTS
FIRST YEAR EXAMINATION IN ARTS (SEMESTER II) – 2017/2018
FND 1206 – INTERMEDIATE MATHEMATICS
(Time: Two Hours)

Answer any five (05) questions

No. of questions: 07

No. of pages: 04

(Each question carries equal marks)

Calculators are not permitted

01. (a) State whether the following numbers are rational or irrational.

$$x = \frac{17}{\sqrt{100}} + \frac{3}{10} \quad (2 \text{ marks})$$

$$y = \frac{17}{\sqrt{10}} + \frac{3}{\sqrt{10}} \quad (3 \text{ marks})$$

(b) Simplify.

$$a = \left\{ \left(\frac{\sqrt{2} + \sqrt{3} + \sqrt{4}}{\sqrt{5} + \sqrt{6} + \sqrt{7}} \right)^0 \right\}^2 \quad (5 \text{ marks})$$

$$b = \frac{\sqrt[4]{16} \times (2)^{-1}}{(\sqrt[3]{3})^3 \times 2^2} \quad (5 \text{ marks})$$

(c) Rationalize the denominator and simplify.

$$\frac{21}{5 - \sqrt{4}} \quad (5 \text{ marks})$$

(Total: 20 marks)

02. (a) Solve the following quadratic equations.

(i) $x^2 - 9x + 14 = 0$ (2 marks)

(ii) $x^2 - 2x - 2 = 0$ (3 marks)

(b) If $x:y = 2:3$, then find $2x + y : 3x - y$ (3 marks)

(c) Solve for x .

$2 \log_2 x - 2 \log_2 7 = \log_2 8 - \log_2 2$ (5 marks)

(d) (i) Solve the equation.

$|x + 1| = 2018$ (3 marks)

(ii) Solve the following simultaneous equations.

$4x - y = 8$

$7x + 2y = 29$ (4 marks)

(Total: 20 marks)

03. (a) Let ϵ (the universal set) = $\{a, b, c, \dots, x, y, z\}$, $X = \{c, a, s, h\}$, $Y = \{h, a, b, i, t\}$,
and $Z = \{t, a, x, i\}$

(i) Find the sets $X \cap Y \cap Z$, $Y \cap Z$, $X \cup Y$, $X \cup Z$, and $(X \cup Y) \cap (X \cup Z)$.

(5 marks)

(ii) Verify that $n(X \cap Y \cap Z) + n[(X \cup Y) \cap (X \cup Z)] = n(X) + n(Y \cap Z)$ for the sets X, Y and Z . (5 marks)

(b) Consider the random experiment of tossing two dice at once.

(i) What is the probability of getting an even number at the first die? (3 marks)

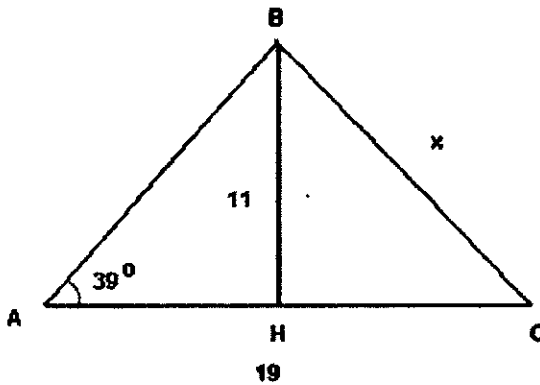
(ii) What is the probability of getting a total of 8? (3 marks)

(iii) Find the probability of getting an even number at the first die or a total of 8?

(Hint: Use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$) (4 marks)

(Total: 20 marks)

04. (a) In the given figure BH is perpendicular to AC. Find x (the length of BC).



(10 marks)

(b) From the top of a 200 meters high building, the angle of depression to the bottom of a second building is 20 degrees. From the same point, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.

(10 marks)

(Total: 20 marks)

05. (a) Find the length of the line segment joining (a, b) and $(-b, a)$. (3 marks)

(b) Vertices of a triangle are given by $A = (3,1)$, $B = (7,11)$ and $C = (-1,5)$. Find midpoints of AB, and AC. Find the equation of line joining the points B and C.

(7 marks)

(c) The equation of the straight line having gradient m and y-intercept c can be written in the form

$$y = mx + c$$

(i) Find the equation of the straight line whose gradient is 5 and y-intercept is 0.

(4 marks)

(ii) Find the equation of the straight line passing through (2,1) and parallel to

$$y = 4x - 5$$

(6 marks)

(Total: 20 marks)

06. (a) If $A = \begin{pmatrix} 1 & -3 & 5 \\ 7 & 1 & 1 \\ -2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 & 1 \\ 2 & 9 & 2 \\ 3 & 6 & 4 \end{pmatrix}$, then find $2A+B$ and $A+2B$.

(5 marks)

(b) Let $X = \begin{pmatrix} 1 & -1 \\ 7 & 4 \end{pmatrix}$, and $Y = \begin{pmatrix} 3 & 8 \\ -3 & 1 \end{pmatrix}$

(i) Find $3X + 4Y$.

(3 marks)

(ii) Show that $(3X + 4Y)^T = 3X^T + 4Y^T$

(5 marks)

(iii) If $Z = (3X + 4Y)^T + \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$, then find Z .

(7 marks)

(Total: 20 marks)

07. (i) Evaluate the following limits.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

(2 marks)

(b) $\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x - 1}$

(3 marks)

(ii) Let $f(x) = 4x^4 + 3x^3 + 2x^2 + x$ and $g(x) = (x - 1)(x - 2)$

(a) Find the derivative of f and the derivative of g .

(5 marks)

(b) Find the derivative of $f \cdot g$ given that $(fg)' = f'g + g'f$.

(5 marks)

(c) What is the value of $(fg)'$ when $x=1$?

(5 marks)

(Total: 20 marks)

Table of Natural Sines (0° to 90°) with columns for degrees (0', 10', 20', 30', 40', 50', 60') and minutes (1' to 9'). Includes sub-headers 'மேல்புறக் குறிப்புகள்' and 'இடை வித்தியாசங்கள் Mean Differences'.

Table of Natural Cosines (0° to 90°) with columns for degrees (0', 10', 20', 30', 40', 50', 60') and minutes (1' to 9'). Includes sub-headers 'மேல்புறக் குறிப்புகள்' and 'இடை வித்தியாசங்கள் Mean Differences'.

